Winners in social ranking under incomplete knowledge: elicitation for the lex-cel

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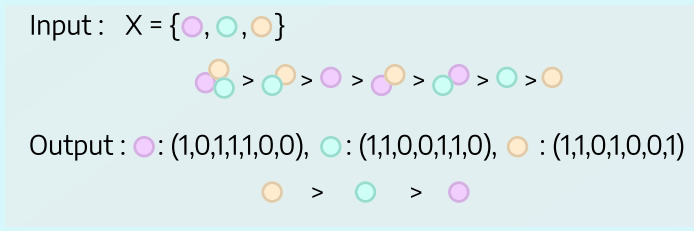
THE SOCIAL RANKING PROBLEM

Given a population *X* and a ranking *R* over all elements of its powerset *P(X)*, the **social ranking problem** consists in determining a ranking over the elements of the population, in a manner that accounts for their impact or influence within all subgroups of the population.

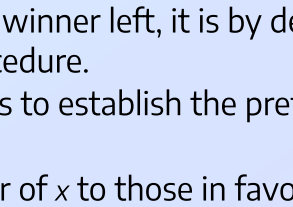


The **lexicographical excellence (lex-cel)** method for social ranking consists in a lexicographical comparison of occurrence vectors associated to each individual of the population.

The size of this problem's input grows exponentially with that of the population. This is both impractical and cognitively demanding of the decider. We therefore choose to focus on a scenario where our knowledge over the full ranking is fragmentary.

THE SOCIAL RANKING PROBLEM UNDER INCOMPLETE KNOWLEDGE

Our input consists in a set of partial rankings (subrankings) compatible with the total ranking *R*.



, , are all possible winners

There is no necessary winner

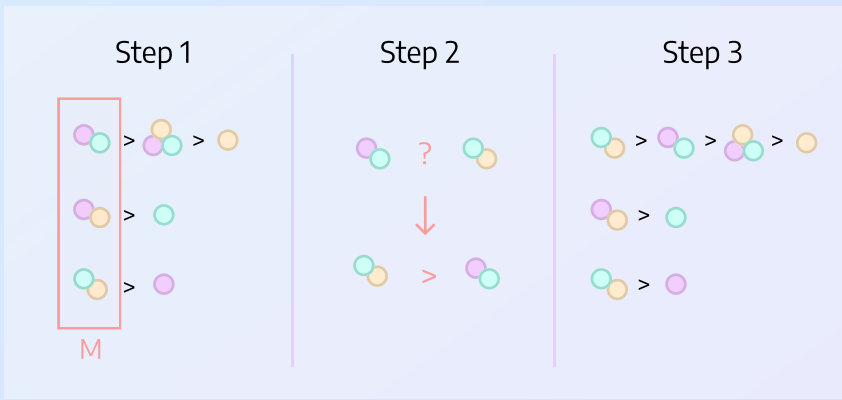
A **possible winner** is an individual *x* such that there exists a completion of the piecemeal order where *x* wins
A **necessary winner** is an individual *x* such that every possible completion of the piecemeal order yields *x* as a winner

Given a set of subrankings over *P(X)*, we try to elicitate the necessary winner according to the lex-cel by presenting relative comparisons to the decider. We explore two approaches.

ELICITATION BY RECONSTITUTION

We try and reconstitute the order over sufficiently many of the top coalitions in order for the lex-cel to be able to determine the winner.

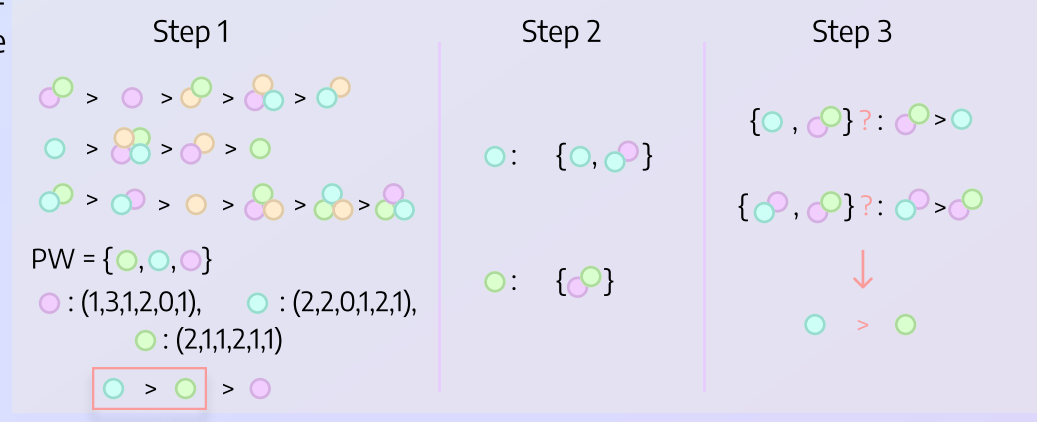
1. Determine the set M of candidate coalitions.
   If there is only one candidate, it is the next in our reconstitution of the total order. We remove it from consideration and add it to our reconstitution. If the reconstitution is enough to determine a winner, we stop.
2. Determine two coalitions to compare between the candidates
   We introduce three variants for step 2: the random (RAND), lexicographic (LEX) and minimal nonempty intersection (MIN_INT) variants.
3. Query the decider and update the subrankings to take into account new knowledge.
   Start back at step 1.



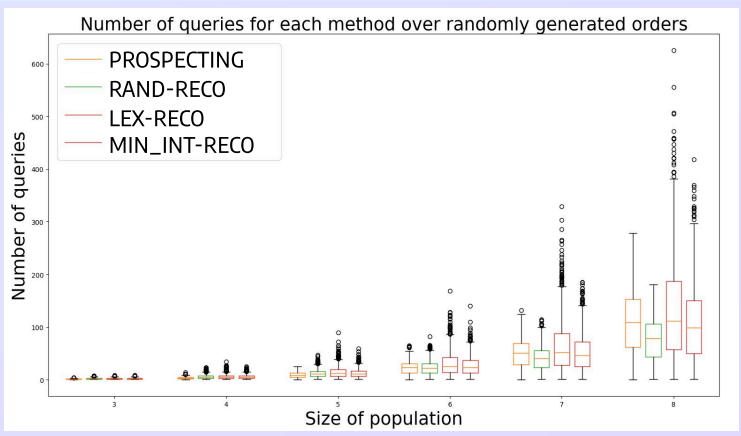
ELICITATION THROUGH PROSPECTING

We determine key coalitions in the comparison of two individuals, and submit targeted queries.

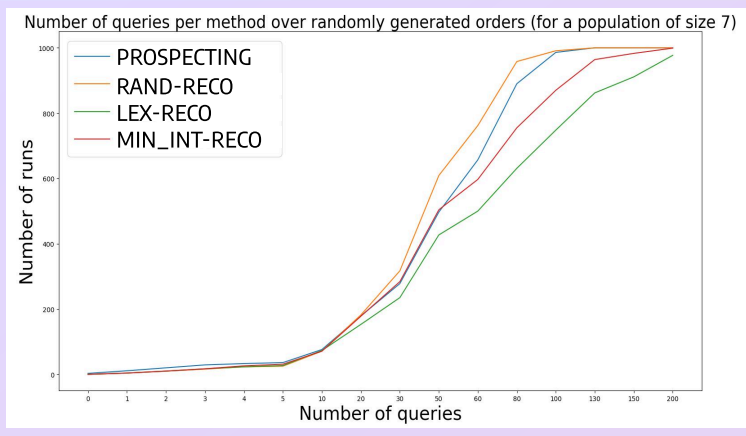
1. Determine the two "most likely" possible winners *x* and *y* among the non-dominated individuals of the population using a generalized version of the lex-cel.
   If there is only one possible winner left, it is by definition the necessary winner, so we stop the procedure.
2. Determine the key coalitions to establish the preference relation over *x* and *y*
3. Compare coalitions in favour of *x* to those in favour of *y*. Once a coalition in favour of *x* is preferred to all those in favour of *y*, we know that *x* is preferred to *y*. Remove *y* from consideration and update the subrankings.
   Start back at step 1.

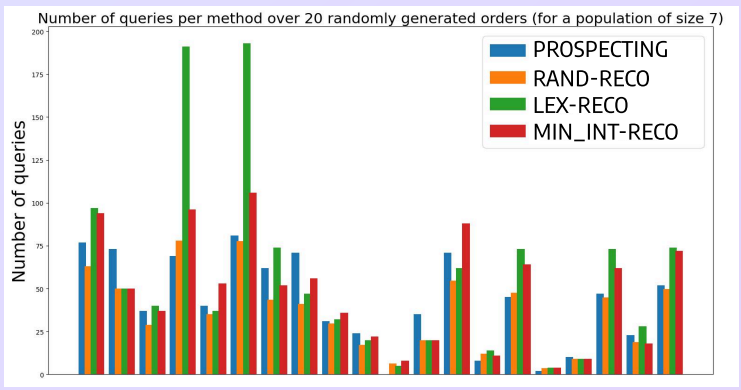


SOME EXPERIMENTAL RESULTS



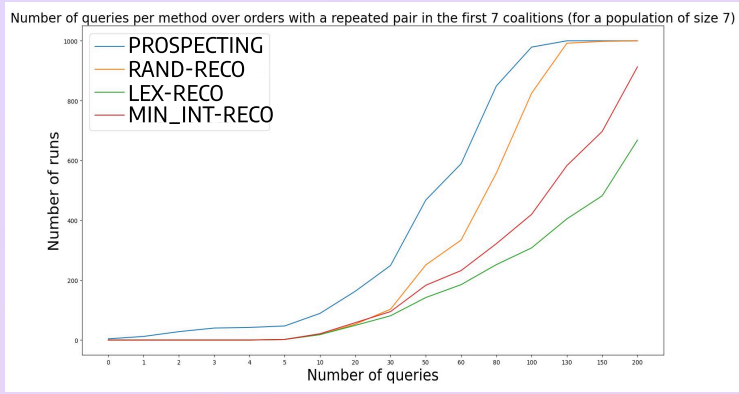
In a general scenario, the number of queries submitted to the user by the **prospecting method** is lower or relatively close to that of the reconstitution method **for smaller populations**. Starting at a population of size 8, however, the random variant of the reconstitution method seems to outperform all other approaches.





On a case-by-case study, nonetheless, it appears the **prospecting method** is able to **discern when no query is necessary** in order to determine the winner, when all variants of the reconstitution method always submit at least one query to the user.

For certain kinds of orders, such as orders where a pair of elements *x*,*y* is present in the first *n* coalitions in the order, the prospecting method is significantly more performant than any variant of the reconstitution method.



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