PASQAL

Solving Maximum Independent Set using Analog Quantum Computing

Pierre Cazals
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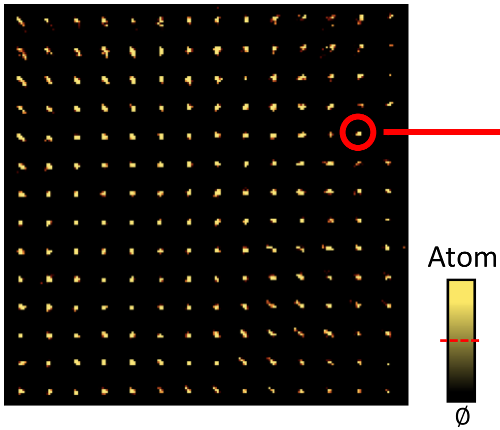
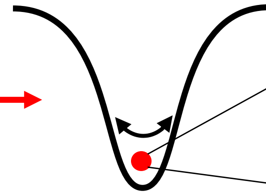
Neutral atoms QPU

- Programmable & scalable

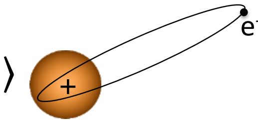




1 atom trapped per tweezer

Rydberg state

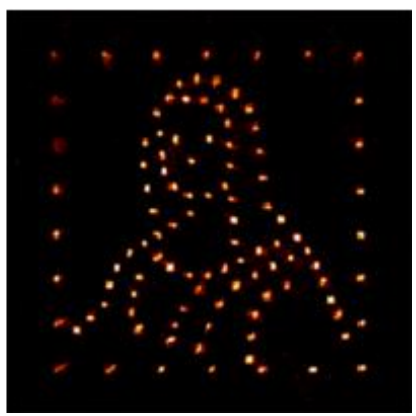
$|r\rangle \rightarrow$ Excited, $|1\rangle$



$|g\rangle \rightarrow$ Not excited, $|0\rangle$

Ground state

Arbitrary configuration



2 routes for quantum computing

There are two ways of realizing quantum calculations

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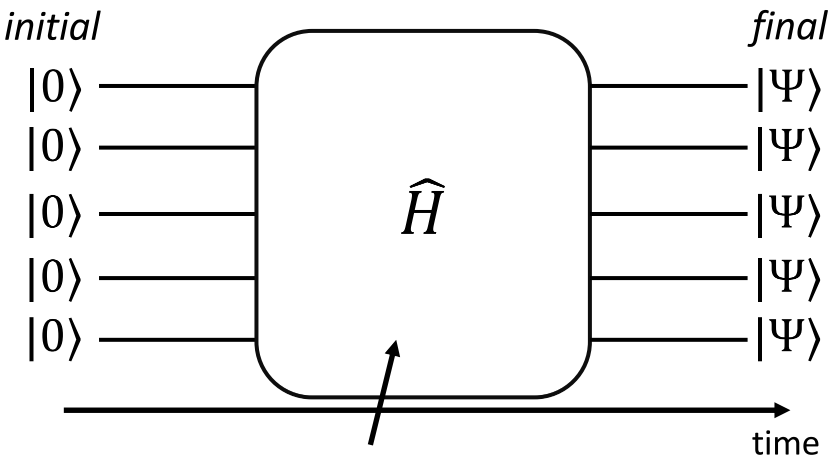
There are two ways of performing quantum calculations

Analog quantum computing

The calculation happens all at once



In practice, this is hitting the system with a single laser

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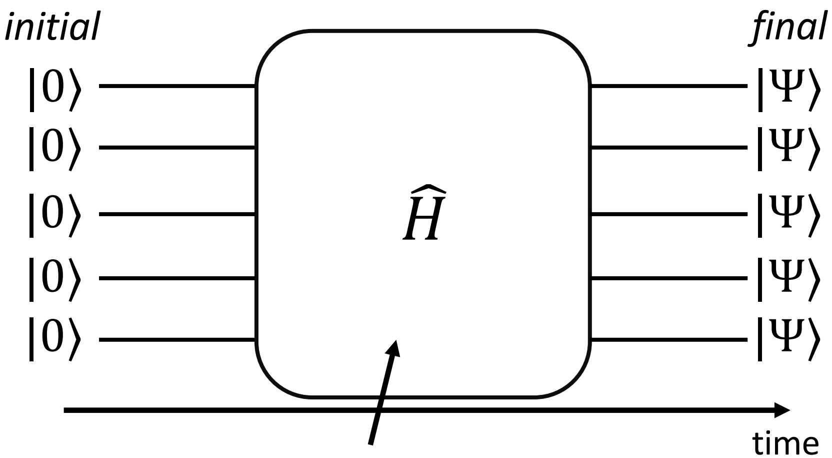

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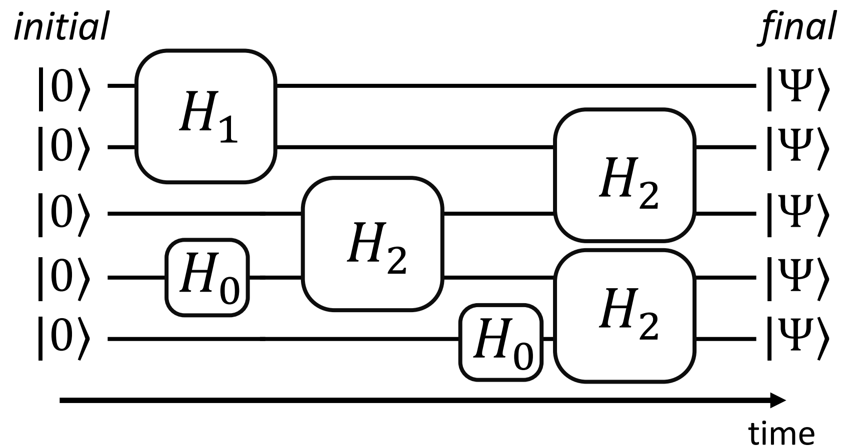
Digital quantum computing

The calculation is digitized into elementary blocks

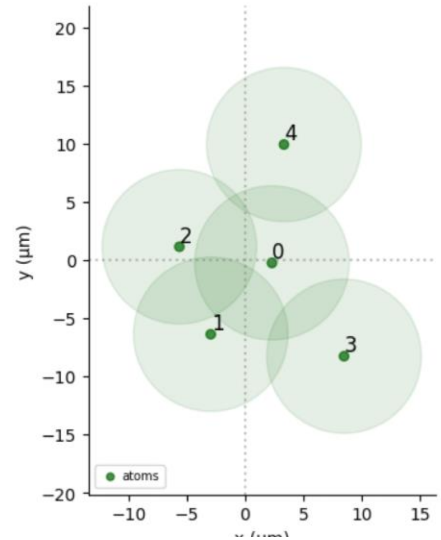


H_0, H_1, H_2 are elementary operations called **gates**

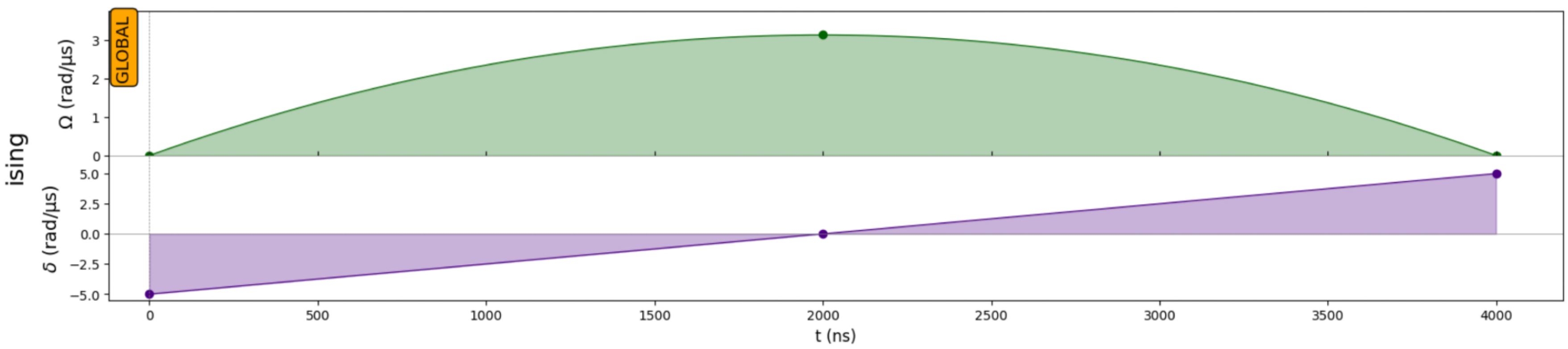
Also hybrid approaches exist…

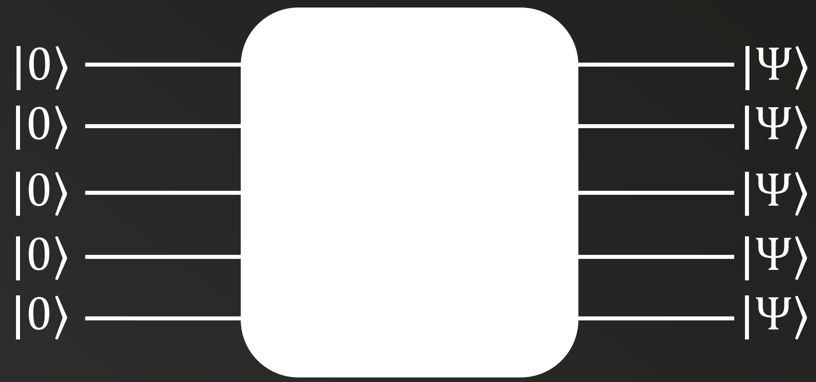
A controllable quantum Hamiltonian

$$H_{\text{Ising}} = \frac{\hbar\Omega(t)}{2}\sum_{i=1}^{N}\hat{\sigma}_i^x - \hbar\delta(t)\sum_{i=1}^{N}\hat{n}_i + \sum_{i<j}\frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$



For a given amplitude, if two atoms are placed close enough, they cannot be excited simultaneously.





How to encode a Maximum Independent Set Problem?

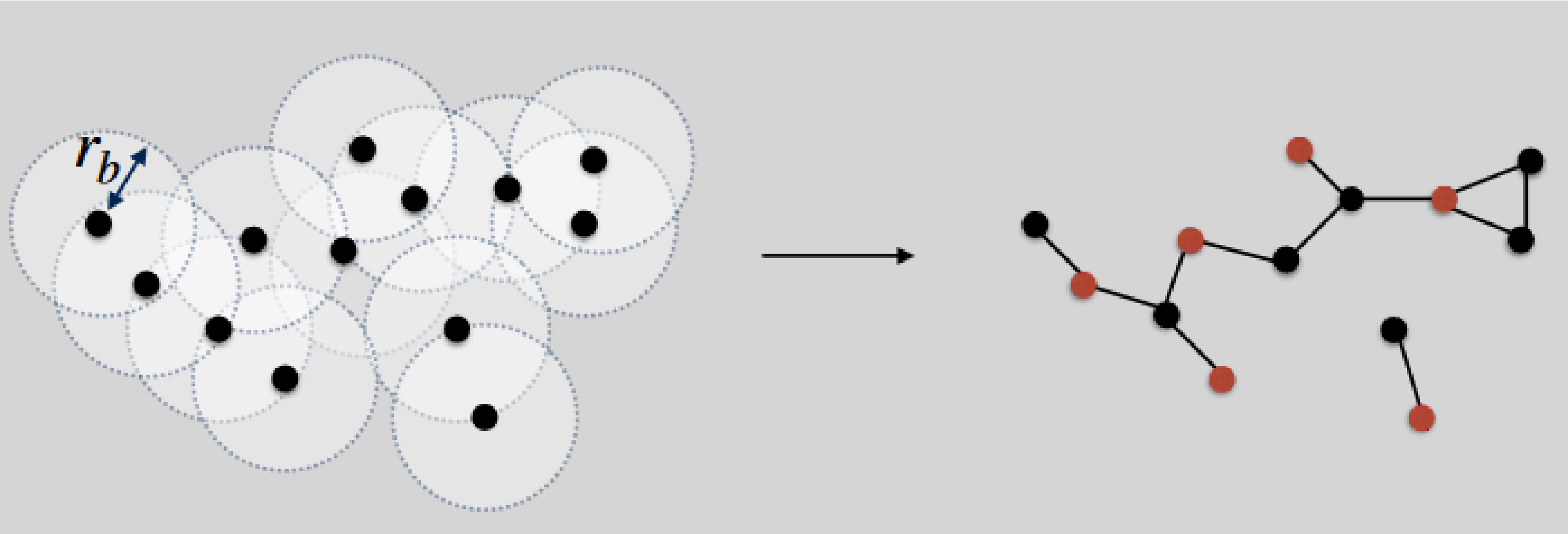
Maximum Independent Set Problem on Unit Disk Graph

Unit-Disk Graph: A graph $G = (V, E)$
where two nodes are connected if their distance is below a fixed threshold r_b

Independent Set (IS): $S \subseteq V / \forall (x, y) \in S^2, (x, y) \notin E$

Maximum Independent Set (MIS): Finding the IS of maximum cardinality

NP-Complete



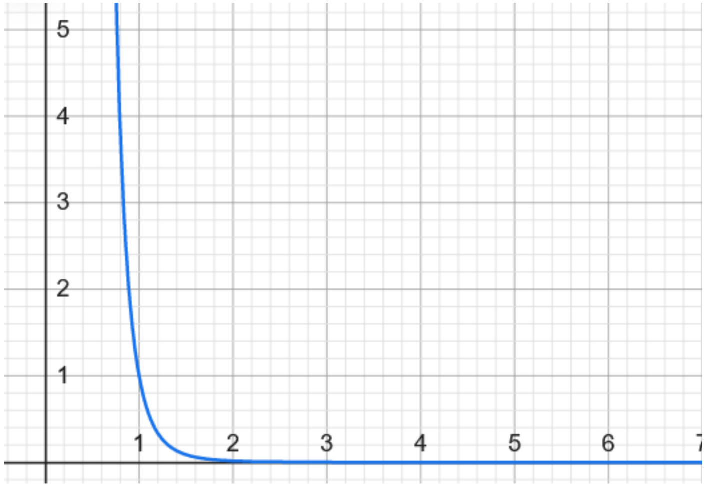
Encoding

Correspondence between the cost function and an Ising Hamiltonian

$$C(z_1, \ldots, z_N) = -\sum_{i=1}^{N} z_i + U \sum_{\langle i,j \rangle} z_i z_j$$

$$H_{ising} = -\hbar\delta \sum_{i=1}^{N} \sigma_z^{(i)} + \sum_{j<i} \frac{C_6}{r_{ij}^6} n_i n_j,$$

Interaction is continuous but strongly decreasing



$$y = \frac{1}{x^6}$$

Solving

We can't encode the final state directly, we must start the process in a state where we know the Hamiltonian and the ground state. Then we continuously evolve the system to converge on the problem's encoding Hamiltonian.

Consequence of the Adiabatic theorem: If we evolve slowly enough, we stay within the groundstates of successive Hamiltonians.

Problem: The distance to the problem's encoding Hamiltonian depends on the chosen evolution.

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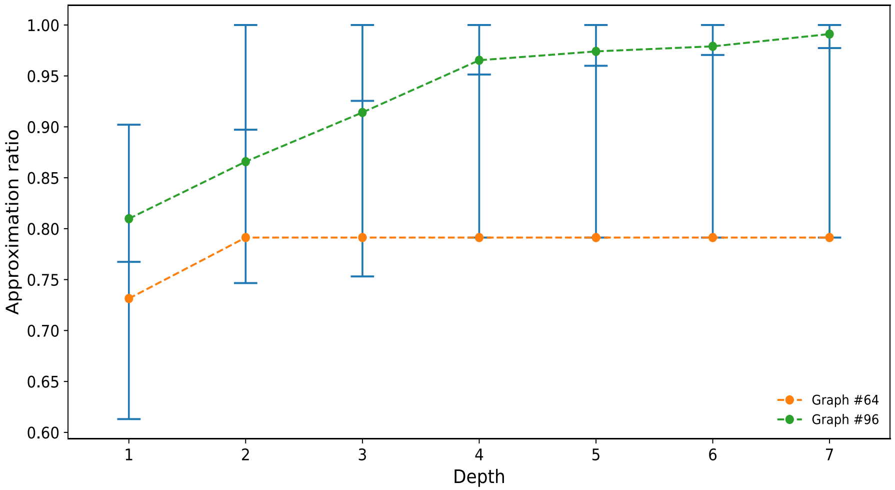
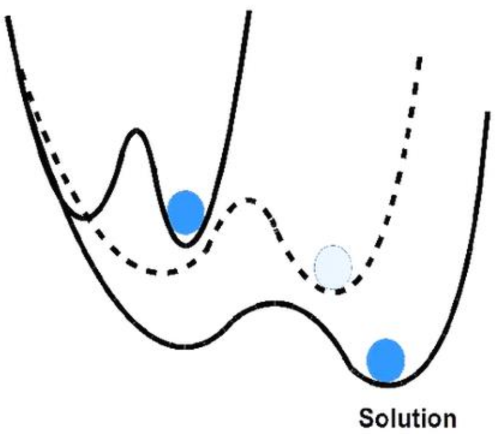
Solving

QAOA (approximate)	Adiabatic Sweep (exact)





Reduction using gadget: encode more general graphs

MIS is NP-complete on Unit Disk graph → Any problem in NP is polynomially encodable.

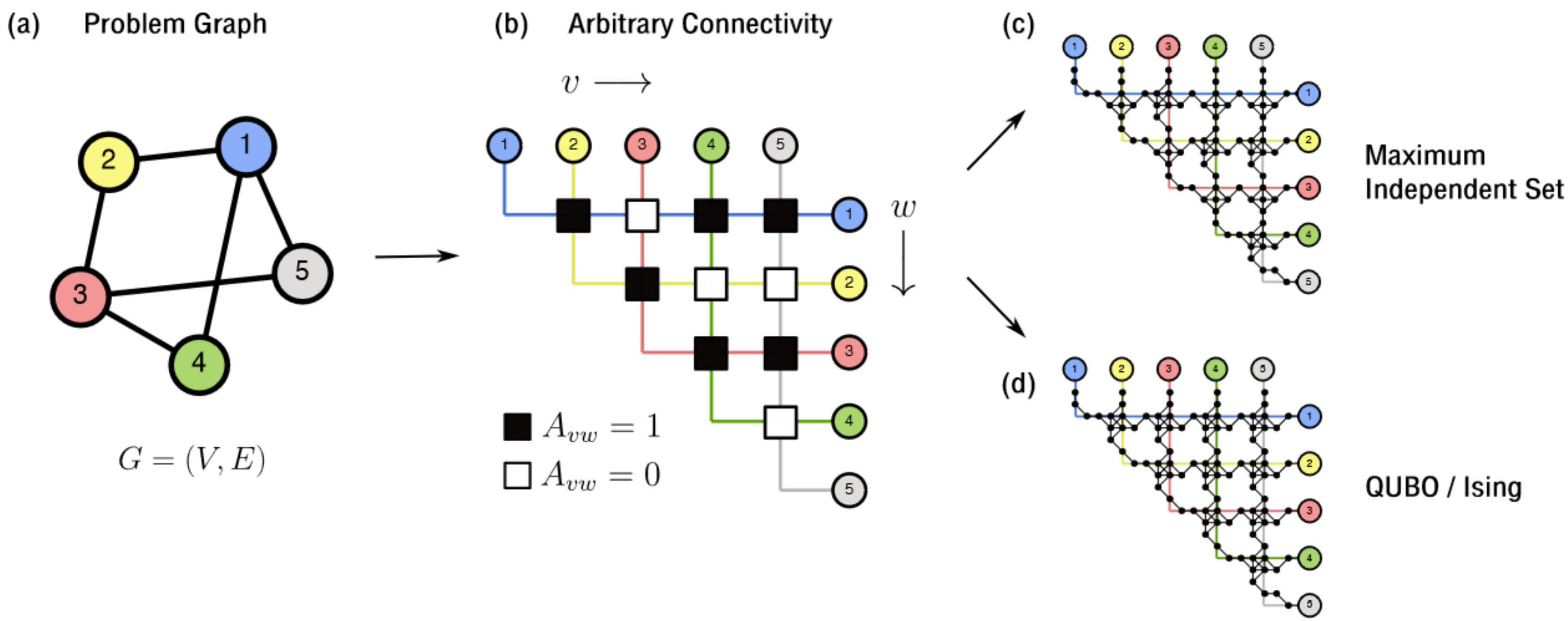


Figure adapted from "Quantum optimization with arbitrary connectivity using Rydberg atom arrays", PRX Quantum, Nguyen et al., 2022