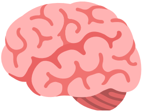
Learning context invariant representations for EEG data

Thibault de Surrel

LAMSADE Spring School - April 17th 2024

Motivation - Brain Computer Interfaces (BCI)



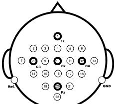
Translate the electrical signals

from the brain into commands

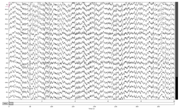
Applications

- Control prosthesis
- Write using a virtual keyboard
- Study the sleep level
- Many more applications...

The data - Electroencephalogram (EEG)



A cap equipped with sensors

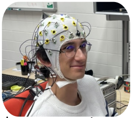


Electroencephalogram (EEG)

$$X = \begin{pmatrix} a_{1,1} & a_{1,2} & & & \\ a_{1,2} & a_{2,2} & & & \\ & & \ddots & & \\ & & & a_{n-1,n-1} & a_{n-1,n} \\ & & & a_{n-1,n} & a_{n,n} \end{pmatrix}$$

Covariance matrix

The data - Electroencephalogram (EEG)



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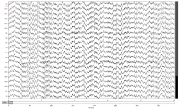

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Covariance matrix

The data - From EEG to covariance matrices

$$\mathbf{E} \in \mathbb{R}^{c \times t} \longrightarrow \mathrm{Cov}(\mathbf{E}) = \frac{1}{1-t} \sum_{i=1}^{t} \mathbf{E}_i \mathbf{E}_i^\top$$

The data - From EEG to covariance matrices

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The data - From EEG to covariance matrices

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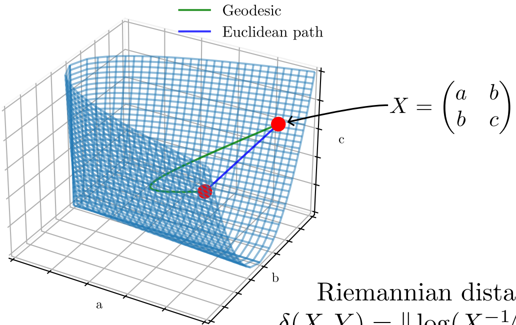
$$\in \mathbb{S}_c^{++} = \begin{Bmatrix} X \in \mathbb{R}^{c \times c} \colon X = X^\top, \\ \forall u \in \mathbb{R}^c \setminus \{0\},\ u^\top X u > 0 \end{Bmatrix}$$



Riemannian distance:

$$\delta(X, Y) = \| \log(X^{-1/2} Y X^{-1/2}) \|_F$$

The limitations

A lot of variabilities

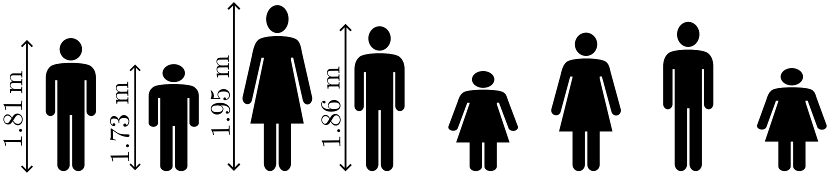
- Intrasubjet : noise, mental state, the subject's state of fatigue...
- Intersubjet

The problematic

How to build a context invariant representation for EEG data ?

Modelization using probability distributions

An example : Model the height of a population



In France, the mean height of the population is 1.71 m. We can model the height of the french population using the *normal distribution* $\mathcal{N}(\mu, \sigma^2)$.

Modelization using probability distributions

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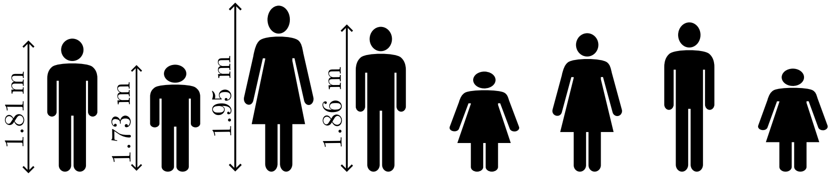

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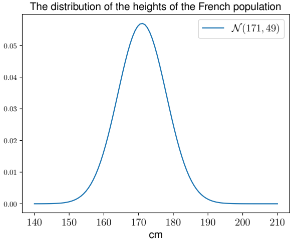

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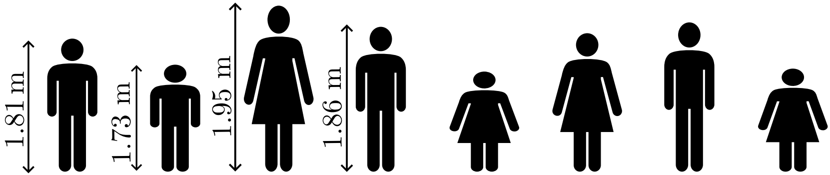

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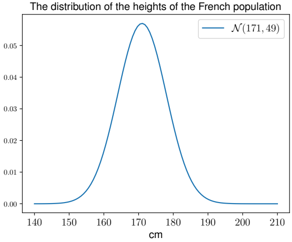
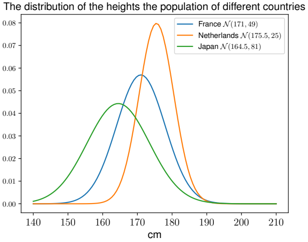
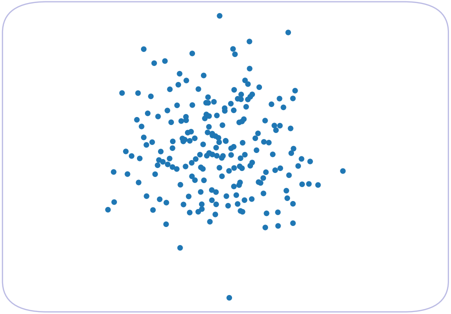




Modelization using probability distributions

Two examples in 2D



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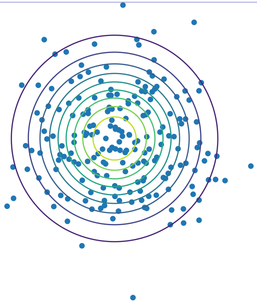

Isotropic gaussian : the spread is uniform.

Modelization using probability distributions

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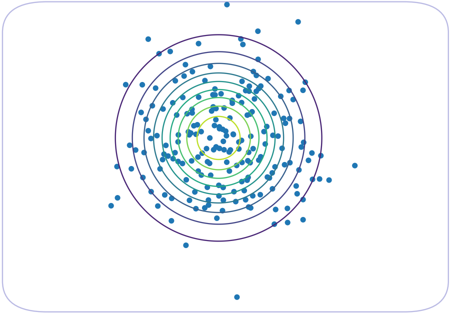

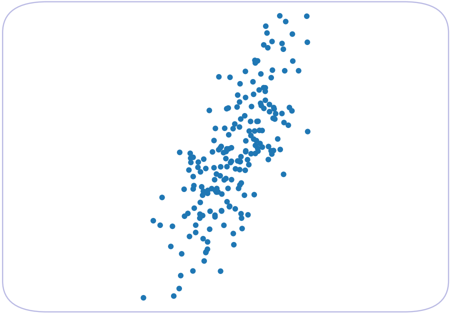

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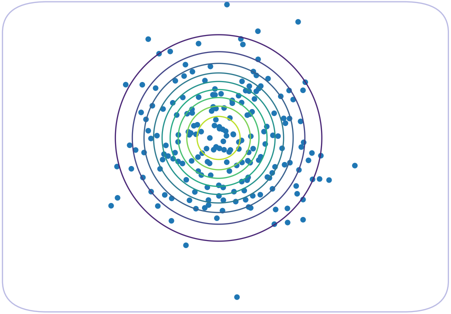

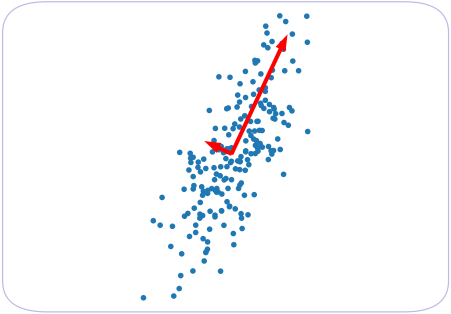

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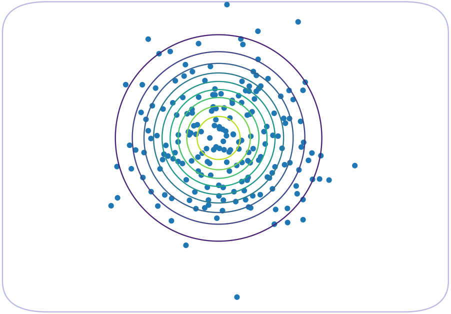

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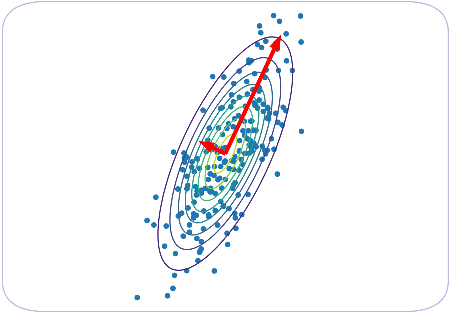
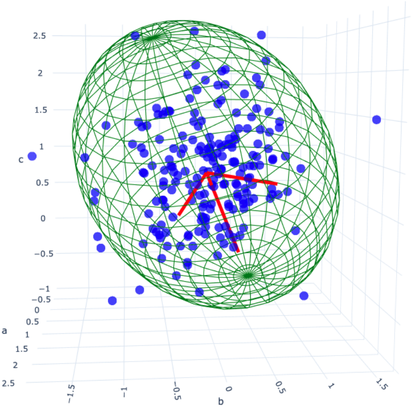


Anisotropic gaussian : there are some preferred directions.

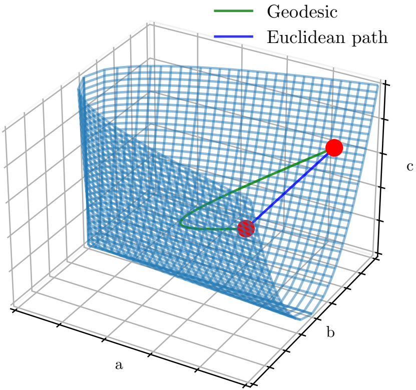
Modelization using probability distributions

One example in 3D



Modelization of the variabilities of EEG

Caution ! We need to take into account the Riemannian geometry of covariance matrices !



2 solutions to send an Euclidean Gaussian onto a Riemannian manifold

- First solution : an *isotropic* gaussian

$$p_{(\mu,\sigma^2)}(x) = \frac{1}{(2\pi\sigma^2)^{d/2}} \exp\left(-\frac{\|x-\mu\|^2}{2\sigma^2}\right)$$

2 solutions to send an Euclidean Gaussian onto a Riemannian manifold

- First solution : an *isotropic* gaussian

$$p_{(\bar{X},\sigma^2)}(X) = \zeta(\sigma)^{-1} \exp\left(-\frac{\delta(X,\bar{X})^2}{2\sigma^2}\right)$$

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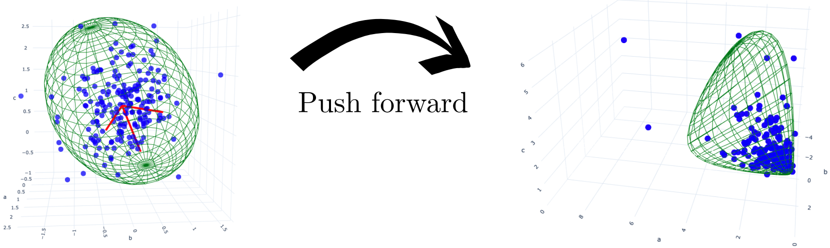
$$p_{(\bar{X},\sigma^2)}(X) = \zeta(\sigma)^{-1} \exp\left(-\frac{\delta(X,\bar{X})^2}{2\sigma^2}\right)$$

- Second solution : an *anisotropic* gaussian

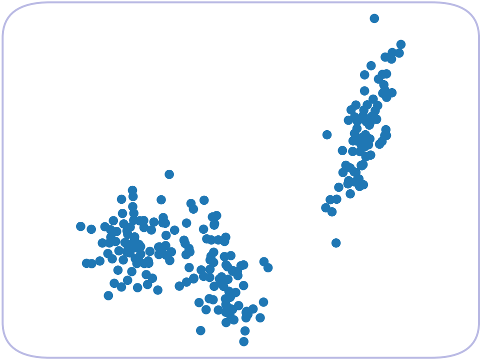


Euclidean anisotropic Gaussian

Riemannian anisotropic Gaussian

Mixture of gaussians

Sometime one gaussian fails at capturing the complexity of the dataset.



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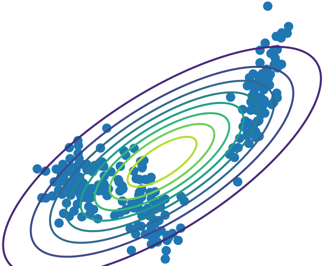

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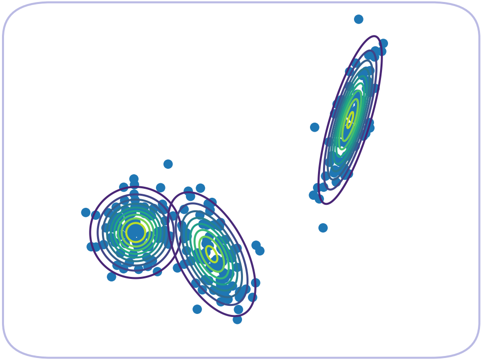

Estimating the parameters of a mixture of gaussians can be done using an *Expectation-Maximization* (EM) algorithm.

The applications

The applications of a Riemannian gaussian in BCI

- Better understand the variabilities
- Build an "informed" classifier
- Detect outliers
- Do some transfer learning
- Do some data augmentation

Conclusion

Motivations

Translate the electrical signals of the brain into commands.

Type of data

Electroencephalogram (multivariate time series) that are then converted into covariance matrices.

Modelization

A probability distribution.

The tools used

Riemannian geometry, probability theory, statistics...

Thank you !
Questions, remarks, comments... ?